Master of Science (M.Sc.) Semester–I (CBCS) (Computer Science) Examination

DISCRETE MATHEMATICAL STRUCTURE

Paper–1

Paper–I

Time : 3 Hours] [Maximum Marks : 80

N.B. :— (1) All questions are compulsory and carry equal marks.

(2) Draw neat and labelled diagrams wherever necessary.

EITHER

1. (a) If A and B are finite sets then $|A \cup B| = |A| + |B| - |A \cap B|$. Define characteristic function and show that :

$f_{A \cap B} = f_A f_B$ and $f_{A \cup B} = f_A + f_B - f_A f_B$ 8

(b) Define Tautology and contradiction. Compute the truth table of $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ 8

OR

(c) If a and b are two positive integers then prove that :

GCD (a,b) . LCM (a,b) = a.b 8

(d) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Show that : $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$ 8

EITHER

2. (a) If R and S are relations from A to B then show that :

(i) $R \subseteq S \Rightarrow R^{-1} \subset S^{-1}$

(ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

(iii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ 8

(b) Find an explicit formula for the sequence defined by :

$C_n = 3C_{n-1} - 2C_{n-2}$

with initial conditions

$C_1 = 5$ and $C_2 = 3$. 8

OR

(c) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be invertible functions then prove that :

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ 8

(d) Let A = {1, 2, 3, 4} and $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Construct the digraph of R and find the relation R on A. 8

EITHER

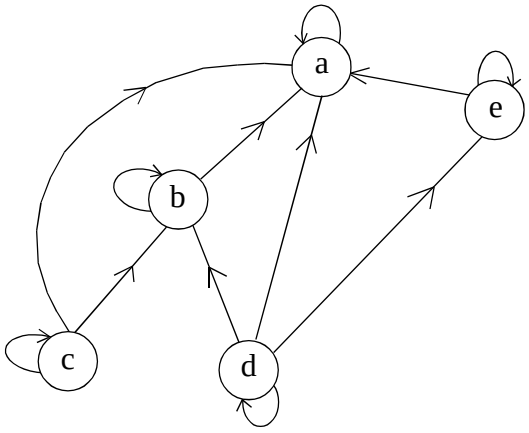
3. (a) Let p (x, y, z) = $(x \wedge y) \vee (y \wedge z')$. Construct the truth table and draw the logic diagram for p. 8

(b) Define Lattice. Prove that if L is a Bounded Distributive Lattice and if its complement exists, then it is unique. 8

OR

(c) Define connected graph. Prove that, if G is a connected graph and every vertex has even degree then there is an Euler Circuit in G. 8

(d) Determine the Hasse diagram and describe the ordered pairs in the relation. Also determine the matrix of partial order of the following digraph : 8



EITHER

4. (a) Let G be the set of all non zero real numbers and let $a * b = \frac{ab}{2}$. Show that (G *) is Abelian. 8

(b) Define semigroup. Let (S, *) and (T, *') be monoids with identities e and e', respectively. Let f : s $\rightarrow$ T be an isomorphism then f(e) = e'. 8

OR

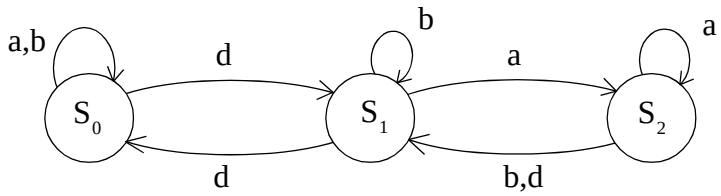
(c) Let G be a group and let a and b be elements of G. Then

(a) $(a^{-1})^{-1} = a$

(b) $(ab)^{-1} = b^{-1} a^{-1}$ 8

(d) Let $S = \{S_0, S_1, S_2\}$ and $I = \{a, b, d\}$. Consider the finite state machine $M = (S, I, F)$ defined by digraph. Compute the functions f_{bad}, f_{add}, and f_{badadd}.

Show that $f_{add} \circ f_{bad} = f_{badadd}$ 8



5. (A) What is the principle of Mathematical Induction ? 4

(B) What is an Equivalence Relation ? 4

(C) What is a Spanning Tree ? Explain Minimal Spanning Tree. 4

(D) What is a Finite State Machine ? 4